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**WEIGHTED ESTIMATES OF DISCRETE HARDY-TYPE
OPERATORS IN SEQUENCE SPACES**

ABSTRACT

**of thesis for the degree of Doctor of Philosophy (PhD)
the specialty 6D060100 –«Mathematics»**

Relevance of the topic. The dissertation is devoted to the problems of obtaining weighted estimates of discrete Hardy-type operators.

One of the main problems in the theory of weighted inequalities is the description of integral and discrete inequalities with the participation of a Hardy-type operator. Currently, in connection with the widespread use of weighted Hardy inequalities, they have become more often considered in harmonic analysis, the theory of differential and difference equations, as well as in other areas of mathematics. Many cases of weighted estimates for linear integral and discrete Hardy-type operators have been considered, and there are a large number of books and scientific articles on this topic in the mathematical literature.

Since the 60s of the last century, weighted estimates of integral and discrete Hardy-type operators, consisting of two weights and two parameters, have become the subject of intensive study of mathematics.

Recently, there has been interest in the study of weighted estimates for Hardy-type operators with three independent weights and three parameters, with their different ratios. For example, such as quasilinear integral or discrete weighted estimates for Hardy operators. This is due to the fact that weighted quasilinear inequalities containing the Hardy integral operator have applications in Morrey-type spaces. In addition, studies are currently underway that show that the characteristics of weighted quasilinear inequalities can be used to study weighted bilinear Hardy-type inequalities.

Note that such weighted estimates for quasilinear integral or discrete operators, which include the Hardy operator, are weighted iterated Hardy-type inequalities. An inequality containing an iteration of the Hardy operator is traditionally considered very difficult to estimate, since it contains three independent weights and three parameters with different ratios. The discrete cases of weighted iterated Hardy-type inequalities have been studied little than their integral counterpart. There is no complete similarity between discrete and integral inequalities. In particular, discrete inequalities have a wider range of research parameters than integral inequalities, and the methods of proof are different.

In mathematics, discrete problems are studied with a delay than integral ones. This is due to the fact that new integral problems have already been solved and are being applied, and the field of application of wider the discrete case is beginning to be intensively investigated. An intensive study of the aforementioned quasilinear integral inequalities shows the importance of studying weighted quasilinear discrete inequalities. For example, in integral analogs of weighted

quasilinear discrete inequalities, the case of the parameter $p \in (0,1]$ is satisfied only if the left-hand side is equal to zero. Therefore, the integral inequalities in this case $p \in (0,1]$ are not considered, but considered when the inequality is discrete.

In mathematics, discrete problems are studied later than integral problems, that is, when new integral problems have already been solved and find their application, the discrete case, which has the widest scope of application, only begins to be intensively studied. An intensive study of the aforementioned quasilinear integral inequalities shows the importance of studying weighted quasilinear discrete inequalities.

The dissertation is devoted to these topical issues, in which the fulfillment of weighted estimates for discrete quasilinear operators containing discrete Hardy operators is considered, i.e. the problems of establishing the fulfillment of discrete quasilinear inequalities with three weights and three parameters are considered.

Integral and discrete Hardy-type inequalities and their various generalized forms are studied in many countries, such as Russia, Sweden, Czech Republic, USA, Croatia, Italy, Norway, and China.

The study of inequalities of Hardy-type in Kazakhstan began with the works of M. Otelbaev and R. Oinarov and was continued in the works of K. Myrzataeva, A. Kalybay, A.M. Temirkhanova, A. Abylaeva, M. Aldai. On the topic of this dissertation work, earlier there were dissertations of S.Kh. Shalginbayeva, A.M. Temirkhanova and Zh.A. Taspaganbetova.

The purpose of the study. To obtain conditions for the fulfillment of discrete quasilinear inequalities with three weights and three parameters containing discrete Hardy operators in sequence spaces, i.e. establishing an estimate for the norm of quasilinear operators on the left-hand side of the inequality using the weighted norm of the sequence on the right-hand side of the inequality.

Research objectives: To achieve the main goal, it is necessary to solve the following problems:

- To obtain in terms of weights, necessary and sufficient conditions for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $1 < p \leq \min\{q, \theta\} < \infty$ and $p \in (0,1]$, $p \leq \min\{q, \theta\}$;
- To obtain in terms of weights, necessary and sufficient conditions for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $0 < q < p \leq \theta$, $p > 1$ and $0 < q < \theta < p$, $p > 1$;
- To obtain in of weights, necessary and sufficient conditions for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $0 < q < p = 1 \leq \theta$ and $0 < q < \theta < p = 1$.

The objects of research. Discrete quasilinear inequalities with three weights and three parameters in the space of sequences containing the discrete Hardy operators.

Research methods. In the dissertation, along with the methods of mathematical and functional analysis, the method partition of sequences "the blocking technique" is used. Various classical inequalities, as well as weighted Hardy inequalities were used in the proof.

Scientific novelty. Criteria for fulfillment of the weighted estimates of quasilinear discrete operators that contains a discrete Hardy operators, i.e. discrete quasilinear inequalities with three weights and three parameters.

Results presented to the defense.

- In terms of weights, necessary and sufficient conditions are obtained for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $1 < p \leq \min\{q, \theta\} < \infty$ and $p \in (0, 1]$, $p \leq \min\{q, \theta\}$;

- In terms of weights, necessary and sufficient conditions are obtained for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $0 < q < p \leq \theta$, $p > 1$ and $0 < q < \theta < p$, $p > 1$;

- In terms of weights, necessary and sufficient conditions are obtained for the fulfillment of weighted estimates of discrete quasilinear operators that contain the discrete Hardy operator, when $0 < q < p = 1 \leq \theta$ and $0 < q < \theta < p = 1$.

The theoretical and practical value of the results. The results of this work are fundamental and make a great contribution to the development of the theory of functional analysis, including the theory of discrete Hardy-type inequalities and estimates of discrete bilinear Hardy operators. The results can be used in functional analysis.

Personal contribution of the applicant. The research work presented in the dissertation was carried out with the direct with author. Criteria for the fulfillment of weighted estimates for quasilinear discrete operators, which include the Hardy operator, i.e. quasilinear inequalities with three weights and three parameters are considered and two-sided estimates for the best positive constants of these inequalities are obtained. The results are published in the form of scientific articles and scientific reports.

Approbation of the dissertation results. The main results of the work presented at the:

- Traditional international scientific conferences of the Institute of Mathematics and Mathematical Modeling (Almaty, 2019 and 2020);
- International Conference "Theoretical and Applied Problems of Mathematics, Mechanics and Informatics" (Karaganda, 2019);
- International Conference "Actual Problems of Analysis, Differential Equations and Algebra" dedicated to the 10th anniversary of the Eurasian Mathematical Journal (EMJ-2019) (Nur-Sultan, 2019);
- 2nd International Scientific and Practical Internet Conference "Integration of Education, Science and Business in the Modern Environment: Summer Debates" (Ukraine, 2020);

Individual results of the dissertation:

- three times performed at the scientific seminar "Functional analysis and its application" (supervisors of the seminar were academicians of NAS RK M.Otelbaev and R. Oinarov, professors E.D. Nursultanov, K.N. Ospanov);
- discussed on the Scientific seminars at Luleå Technical University and at the University of Tromsø – The Arctic University of Norway, Faculty of Mathematics, led by Professor L.E. Persson;

- repeatedly presented and discussed at the scientific seminar "Weighted inequalities and their applications" (supervisors of the seminar were academicians of NAS RK R. Oinarov, associate professors A.M. Temirkhanova, A.M. Abylayeva, associate professor M. Alday).

Publications. 9 works on the topic of the dissertation, including 3 articles in scientific publications included in the list recommended by the Control Committee for Education and Science of the MES RK, 1 article indexed in the database Scopus, Web of Science (Web of Science, Impact factor – 1.25, 2020, Q2) 5 publications in the materials of international scientific conferences, including 1 publication in the materials of foreign international conferences.

The structure and scope of the dissertation. The dissertation consists of an introduction, two chapters, a conclusion and a list of references. The total volume of the dissertation is 72 pages.

In the first chapter, we present well-known statements and lemmas necessary to prove the main results, and also obtain necessary and sufficient conditions for the fulfillment of weighted estimates of a quasilinear discrete operator that contain a discrete Hardy operator under the following parameter conditions:

- $1 < p \leq \min\{q, \theta\} < \infty$ and $p \in (0, 1]$, $p \leq \min\{q, \theta\}$;
- $0 < q < p \leq \theta$, $p \geq 1$ and $0 < q < \theta < p$, $p \geq 1$.

In the second chapter, necessary and sufficient conditions are obtained for the fulfillment of weighted estimates of a quasilinear discrete operator, which contain the discrete conjugate Hardy operator under the following parameter conditions:

- $1 < p \leq \min\{q, \theta\} < \infty$ and $p \in (0, 1]$, $p \leq \min\{q, \theta\}$;
- $0 < q < p \leq \theta$, $p > 1$ and $0 < q < \theta < p$, $p > 1$.

In the conclusion, the main conclusions are formulated and the scope of their application is described. The dissertation ends with a list of used literature.